General Calculus Practice

Use this key to check answers and read the explanations.

1. As x approaches 1 from either side, the graph approaches y = -2. A filled point at (1, 4) gives the actual function value there. What is the limit as x approaches 1?

A. -2
B. 4
C. 1
D. The limit does not exist

Correct answer: A. -2
Both sides approach -2. The limit follows the nearby values, while the filled point gives the separate value at x = 1.

2. The graph of f passes through (4, 8). Its tangent line at that point rises 3 units for every 1 unit to the right. What is f'(4)?

A. 1/3
B. 8
C. 3
D. -3

Correct answer: C. 3
The derivative at x = 4 is the tangent line's slope. Its rise divided by its run is 3/1, or 3.

3. If f(x) = 5x^3 - 7x + 4, what is f'(x)?

A. 15x^2 - 7x
B. 15x^2 - 7
C. 5x^2 - 7
D. 15x^2 - 3

Correct answer: B. 15x^2 - 7
The power rule gives 15x^2 from the cubic. The linear term contributes -7, and the derivative of the constant is zero.

4. The graph of f is horizontal at y = 3 from x = 2 to x = 7. What is the definite integral of f from 2 to 7?

A. 5
B. 9
C. 21
D. 15

Correct answer: D. 15
The region above the x-axis is a rectangle. Its width is 7 - 2 = 5 and its height is 3, so the integral is 15.

5. What is the limit of x^2 + 3x - 2 as x approaches -2?

A. 8
B. -4
C. -2
D. 0

Correct answer: B. -4
The expression is a polynomial, so its limit is its value at -2. Substitution gives 4 - 6 - 2 = -4.

6. What is the limit of (x^2 - 2x - 15)/(x - 5) as x approaches 5?

A. 8
B. 2
C. 0
D. The limit does not exist

Correct answer: A. 8
The numerator factors as (x - 5)(x + 3). For x near 5 with x different from 5, the ratio equals x + 3. That expression approaches 8.

7. For f(x) = (2x - 1)^3, what is f'(3)?

A. 125
B. 75
C. 30
D. 150

Correct answer: D. 150
The chain rule gives f'(x) = 6(2x - 1)^2. At x = 3, the squared expression is 5^2, so the derivative is 150.

8. A differentiable function has f'(x) > 0 just left of x = 4, f'(4) = 0, and f'(x) < 0 just right of x = 4. What does its graph have at x = 4?

A. A corner
B. A local minimum
C. A local maximum
D. A point where it increases through x = 4

Correct answer: C. A local maximum
The derivative changes from positive to negative, so the graph rises into x = 4 and falls after it. That gives a local maximum.

9. What is the definite integral of 2x + 3 from x = 1 to x = 4?

A. 24
B. 28
C. 15
D. 21

Correct answer: A. 24
An antiderivative is x^2 + 3x. Its value at 4 is 28 and its value at 1 is 4. The difference is 24.

10. Which expression gives all antiderivatives of 12x^2 - 4, where C is an arbitrary constant?

A. 24x + C
B. 4x^3 - 4 + C
C. 12x^3 - 4x + C
D. 4x^3 - 4x + C

Correct answer: D. 4x^3 - 4x + C
Differentiating 4x^3 - 4x + C yields 12x^2 - 4. Any two antiderivatives of this polynomial differ by a constant.

11. A graph's tangent slopes are negative throughout an interval, and those slopes increase as x increases. Which description fits the graph on that interval?

A. Increasing and concave up
B. Decreasing and concave down
C. Decreasing and concave up
D. Increasing and concave down

Correct answer: C. Decreasing and concave up
Negative slopes mean the graph is decreasing. The slopes become less negative, so the derivative is increasing and the graph is concave up.

12. A cart's position is s(t) = 2t^3 - 9t^2 + 12t meters for t from 0 to 3 seconds. During which open interval is the cart moving toward smaller positions?

A. Between 0 and 1 seconds
B. Between 1 and 2 seconds
C. Between 2 and 3 seconds
D. Throughout the interval between 0 and 3 seconds

Correct answer: B. Between 1 and 2 seconds
Velocity is s'(t) = 6(t - 1)(t - 2). Between 1 and 2 the two factors have opposite signs, so the velocity is negative. Both other intervals have positive velocity.

13. A tank starts with 10 liters and has no outflow. Its inflow rate falls linearly from 6 to 2 liters per second over the first 4 seconds, then stays at 2 through second 7. How much water is in the tank at second 7?

A. 38 liters
B. 22 liters
C. 28 liters
D. 32 liters

Correct answer: D. 32 liters
The first 4 seconds add 16 liters, using the average rate of 4 liters per second. The next 3 seconds add 6 liters. Adding the initial 10 gives 32 liters.

14. Suppose f'(4) = 7. What is the limit of [f(4 + 3h) - f(4)] / h as h approaches 0?

A. 21
B. 7
C. 7/3
D. 0

Correct answer: A. 21
Let u = 3h. Then the quotient is 3 times [f(4 + u) - f(4)] / u. As u approaches zero, the bracketed difference divided by u approaches f'(4) = 7. Multiplying by 3 gives 21.

15. For f(x) = x^2 - 4 on [0, 3], what are the definite integral and the total geometric area between the curve and the x-axis, in that order?

A. -3 and 7/3
B. 3 and 23/3
C. -3 and 23/3
D. 23/3 and 3

Correct answer: C. -3 and 23/3
The curve crosses at x = 2. Evaluating x^3/3 - 4x gives -16/3 on [0, 2] and 7/3 on [2, 3]. Their sum is -3, and the sum of their magnitudes is 23/3 square units.