Derivatives and Differentiation Practice

Use this key to check answers and read the explanations.

1. Water depth H(t) is measured in centimeters and time t in minutes. If $H'(6) = -0.8$, what does that tell you?

A. At minute 6, the depth is falling at 0.8 cm per minute.
B. In the first six minutes, the depth falls by exactly 0.8 cm.
C. At minute 6, the depth is rising at 0.8 cm per minute.
D. The depth at minute 6 is 0.8 cm.

Correct answer: A. At minute 6, the depth is falling at 0.8 cm per minute.
$H'(6)$ is the instantaneous change in depth per minute at $t = 6$. Its negative sign indicates a falling depth.

2. For $x \neq 0$, what is the derivative of $f(x) = 5/x$?

A. $-5/x$
B. $5/x^2$
C. $-5/x^2$
D. 5

Correct answer: C. $-5/x^2$
Rewrite $5/x$ as $5x^{-1}$. The power rule multiplies by -1 and reduces the exponent to -2, giving $-5/x^2$.

3. For x measured in radians, what is the derivative of $2\cos(x) - 3\sin(x)$?

A. $2\sin(x) - 3\cos(x)$
B. $-2\sin(x) - 3\cos(x)$
C. $-2\sin(x) + 3\cos(x)$
D. $-2\cos(x) - 3\sin(x)$

Correct answer: B. $-2\sin(x) - 3\cos(x)$
The derivative of $\cos(x)$ is $-\sin(x)$. The derivative of $\sin(x)$ is $\cos(x)$. Keep the multipliers 2 and -3.

4. If f(x) = 7e^x + 6, what is f′(x)?

A. 7e^x + 6
B. 7xe^x
C. e^x
D. 7e^x

Correct answer: D. 7e^x
The derivative of e^x is e^x, so its coefficient 7 stays. The constant term 6 has derivative 0.

5. A function equals $x + 3$ for $x < 2$ and $4x - 3$ for $x \geq 2$. What is its derivative at $x = 2$?

A. 1
B. It does not exist
C. 4
D. 5

Correct answer: B. It does not exist
Both pieces meet at $f(2) = 5$. The left-hand slope is 1 and the right-hand slope is 4, so there is no common derivative at 2.

6. At $x = 3$, $a(3) = 4$, $a'(3) = -2$, $b(3) = 5$, and $b'(3) = 7$. If $f(x) = a(x)b(x)$, what is $f'(3)$?

A. 18
B. -14
C. 28
D. -10

Correct answer: A. 18
The product rule gives $a'b + ab'$. At $x = 3$ this is $(-2)(5) + (4)(7) = -10 + 28 = 18$.

7. For $x \neq -2$, what is the derivative of $(4x - 1)/(x + 2)$?

A. $9/(x + 2)$
B. $-9/(x + 2)^2$
C. 4
D. $9/(x + 2)^2$

Correct answer: D. $9/(x + 2)^2$
The quotient rule gives $[4(x + 2) - (4x - 1)]/(x + 2)^2$. The numerator simplifies to 9.

8. For $x < 4$, what is the derivative of $\sqrt{(8 - 2x)}$?

A. $1/[2\sqrt{(8 - 2x)}]$
B. $1/\sqrt{(8 - 2x)}$
C. $-1/\sqrt{(8 - 2x)}$
D. $-2/\sqrt{(8 - 2x)}$

Correct answer: C. $-1/\sqrt{(8 - 2x)}$
A square root has derivative $1/(2\sqrt{u})$ with respect to its input u. The input $8 - 2x$ has derivative -2, so their product is $-1/\sqrt{(8 - 2x)}$.

9. If $h(t) = 5t^4 - 14t^2 + 8$, what is $h''(1)$?

A. 32
B. -8
C. 46
D. 60

Correct answer: A. 32
First $h'(t) = 20t^3 - 28t$. Differentiate again to get $h''(t) = 60t^2 - 28$. At $t = 1$ that equals 32.

10. A differentiable, one-to-one function g satisfies g(2) = 7 and g′(2) = −4. What is the derivative of its inverse function at input 7?

A. −1/7
B. 1/4
C. −4
D. −1/4

Correct answer: D. −1/4
The inverse sends 7 back to 2. Its derivative at 7 is the reciprocal of g′(2), which is 1/(−4).

11. A function equals $x^2 + ax$ when $x \leq 1$ and $8x + b$ when $x > 1$. Which pair (a, b) makes the function differentiable at x = 1?

A. (8, 1)
B. (6, 1)
C. (6, −1)
D. (4, −3)

Correct answer: C. (6, −1)
Match slopes to get 2 + a = 8, so a = 6. Match values to get 1 + 6 = 8 + b, so b = −1. The joined pieces have both the same value and the same slope.

12. A curve satisfies $x^2 + xy + 2y^2 = 28$. What is dy/dx at the point (2, 3)?

A. 1/2
B. −1/2
C. −2
D. −7/12

Correct answer: B. −1/2
Differentiate with y as a function of x to get 2x + y + (x + 4y)dy/dx = 0. At (2, 3), this is 7 + 14dy/dx = 0, so dy/dx = −1/2.

13. A rectangle's area is increasing at 5 m²/min. At one instant its length is 9 m and its width is 6 m, shrinking at 1/3 m/min. How fast is its length changing at that instant?

A. Increasing at 8/9 m/min
B. Increasing at 1/3 m/min
C. Increasing at 5/6 m/min
D. Increasing at 4/3 m/min

Correct answer: D. Increasing at 4/3 m/min
Differentiate area = length × width. If L′ is the length rate, then 5 = 6L′ + 9(−1/3). Solving gives L′ = 8/6 = 4/3 m/min.

14. Let $f(x) = x^2 \ln(1 + 5x)$ for $x > -1/5$. What is $f'(1)$?

A. $2 \ln(6) + 5/6$
B. $2 \ln(6) + 1/6$
C. $2 \ln(6)$
D. $5/6$

Correct answer: A. $2 \ln(6) + 5/6$
The product rule gives $2x \ln(1 + 5x) + x^2 \times 5/(1 + 5x)$. At $x = 1$, those terms are $2 \ln(6)$ and $5/6$.

15. Let $F(x) = (x + 4)^{(3x)}$ for $x > -4$. Which expression equals $F'(x)$ throughout that domain?

A. $3(x + 4)^{(3x)} \ln(x + 4)$
B. $(x + 4)^{(3x)} [3 \ln(x + 4) + 3/(x + 4)]$
C. $(x + 4)^{(3x)} [3 \ln(x + 4) + 3x/(x + 4)]$
D. $3x(x + 4)^{(3x - 1)}$

Correct answer: C. $(x + 4)^{(3x)} [3 \ln(x + 4) + 3x/(x + 4)]$
Take natural logs to get $\ln(F(x)) = 3x \ln(x + 4)$. The product rule gives $F'(x)/F(x) = 3 \ln(x + 4) + 3x/(x + 4)$. Multiply by $F(x)$.