Transformations of Functions

Use this key to check answers and read the explanations.

1. What happens to the graph of y = f(x) when it becomes y = f(x) + 7?
   A. It shifts up 7 units.
   B. It shifts down 7 units.
   C. It shifts left 7 units.
   D. It shifts right 7 units.
   Correct answer: A. It shifts up 7 units.
   Adding 7 after evaluating the function raises each output by 7 at the same input.

2. How does the graph of y = f(x − 3) compare with the graph of y = f(x)?
   A. It shifts up 3 units.
   B. It shifts left 3 units.
   C. It shifts right 3 units.
   D. It shifts down 3 units.
   Correct answer: C. It shifts right 3 units.
   An old input x is reached at the new input x + 3. Each point moves right.

3. Across which axis is the graph reflected when y = f(x) becomes y = −f(x)?
   A. The y-axis
   B. The x-axis
   C. The line y = x
   D. The line y = 1
   Correct answer: B. The x-axis
   Each output changes sign and each input stays the same. That reflects the graph across the x-axis.

4. Which change produces the graph of y = f(−x) from y = f(x)?
   A. A shift down 1 unit
   B. Reflection across the x-axis
   C. A vertical stretch by 2
   D. Reflection across the y-axis
   Correct answer: D. Reflection across the y-axis
   Each input changes sign for the corresponding output. The graph reflects across the y-axis.

5. What does multiplying the function by 3, as in $y = 3f(x)$, do to its graph?

A. It stretches the graph horizontally by a factor of 3.
B. It stretches the graph vertically by a factor of 3.
C. It shifts the graph up 3 units.
D. It compresses the graph vertically to one third of its height.

Correct answer: B. It stretches the graph vertically by a factor of 3.
The input is unchanged and each output is multiplied by 3.

6. What is the horizontal scale factor from the graph of $f(x)$ to the graph of $f(4x)$?

A. 1/4
B. 4
C. 1/16
D. 16

Correct answer: A. 1/4
Each old input is reached at one quarter of its old horizontal coordinate. The graph compresses toward the y-axis.

7. Suppose $f(2) = -5$ and $g(x) = f(x + 4) + 3$. What is $g(-2)$?

A. 8
B. -8
C. 2
D. -2

Correct answer: D. -2
The input to f is $-2 + 4 = 2$. Its output is -5, and adding 3 gives -2.

8. The point $(-3, 4)$ lies on $y = f(x)$. What is its corresponding point on $y = f(x - 5) - 2$?

A. (2, 6)
B. $(-8, 2)$
C. (2, 2)
D. $(-3, -3)$

Correct answer: C. (2, 2)
The graph shifts right 5 units and down 2 units. The point moves to $(-3 + 5, 4 - 2) = (2, 2)$.

9. Where is the vertex of $g(x) = (x + 2)^2 - 4$?

A. $(-2, -4)$
B. $(2, -4)$
C. $(-2, 4)$
D. (2, 4)

Correct answer: A. $(-2, -4)$
The squared term is zero at $x = -2$. The graph is the square function shifted left 2 and down 4.

10. Suppose the range of f is all values from −1 through 8. What is the range of g(x) = 2f(x) − 3?

A. 1 through 19
B. −1 through 16
C. −4 through 5
D. −5 through 13

Correct answer: D. −5 through 13
Apply 2y − 3 to the endpoints. The lower endpoint becomes −5, and the upper endpoint becomes 13.

11. If (6, −4) lies on y = f(x), which point must lie on y = −2f(x − 1) + 3?

A. (7, −11)
B. (5, 11)
C. (7, 11)
D. (6, 5)

Correct answer: C. (7, 11)
The old input 6 occurs when x − 1 = 6, so x = 7. The transformed output is −2(−4) + 3 = 11.

12. Suppose f(3) = 7 and g(x) = f(6 − 2x) − 4. Which point must be on the graph of g?

A. (−1.5, 3)
B. (1.5, 3)
C. (4.5, 11)
D. (3, 7)

Correct answer: B. (1.5, 3)
Solve 6 − 2x = 3 to get x = 1.5. At that input, the output is 7 − 4 = 3.

13. The domain of f is all x from −4 through 8. What is the domain of g(x) = f(2x − 4)?

A. −12 through 12
B. −4 through 8
C. −8 through 16
D. 0 through 6

Correct answer: D. 0 through 6
The inside input must satisfy −4 ≤ 2x − 4 ≤ 8. Add 4 and divide by 2 to obtain 0 ≤ x ≤ 6.

14. The range of f is all values from 2 through 9. What is the range of g(x) = −3f(x) + 5?

A. −22 through −1
B. −1 through 22
C. −32 through −11
D. 11 through 32

Correct answer: A. −22 through −1
The endpoint 2 maps to −1, and 9 maps to −22. The negative coefficient reverses their order.

15. Points (−1, 0) and (2, 5) lie on f. Points (3, −2) and (6, 13) lie on g(x) = af(x − 4) + k. What are a and k?

A. a = −3 and k = −2
B. a = 3 and k = 2
C. a = 3 and k = −2
D. a = 2 and k = 3

Correct answer: C. a = 3 and k = −2
At x = 3, f(x − 4) is zero, so k = −2. At x = 6, 5a − 2 = 13, giving a = 3.