Integral Calculus Practice

Use this key to check answers and read the explanations.

1. Suppose f is continuous near x = 4 and f(4) = 9. What is the definite integral of f from x = 4 to x = 4?

A. 0
B. 4
C. 9
D. 36

Correct answer: A. 0
The integration interval has zero width, so the definite integral is zero.

2. The definite integral of g from x = 1 to x = 3 is -4. What is the definite integral of g from x = 3 to x = 1?

A. 2
B. -4
C. 4
D. 0

Correct answer: C. 4
Reversing the bounds negates the integral. The opposite of -4 is 4.

3. The definite integral of h from x = -1 to x = 2 is 6, and from x = 2 to x = 5 it is -8. What is the definite integral of h from x = -1 to x = 5?

A. 2
B. -2
C. 14
D. -14

Correct answer: B. -2
The two intervals meet at x = 2, so their definite integrals add. The result is 6 + (-8) = -2.

4. For x > 0, which expression gives all antiderivatives of -8/x^3, with C an arbitrary constant?

A. 4/x^4 + C
B. -4/x^2 + C
C. 8/x^2 + C
D. 4/x^2 + C

Correct answer: D. 4/x^2 + C
Write 4/x^2 as 4x^(-2). Differentiation gives -8x^(-3), which is the required function. Any constant C can be added.

5. The definite integral of f from x = 2 to x = 8 is 27. What is the average value of f on that interval?

A. 3.375
B. 4.5
C. 6
D. 27

Correct answer: B. 4.5
Divide the definite integral by the interval length. The length is 8 - 2 = 6, so the average is 27/6 = 4.5.

6. Estimate the definite integral of f(x) = x^2 + 1 from x = 0 to x = 6 using three equal-width rectangles and right endpoint heights. What estimate do you get?

A. 118
B. 46
C. 76
D. 59

Correct answer: A. 118
Each rectangle has width 2. The right endpoints are 2, 4, and 6, with heights 5, 17, and 37. Multiply their sum by 2 to get 118.

7. A function satisfies F'(x) = 6x^2 - 4x + 1 and F(1) = 7. What is F(3)?

A. 43
B. 39
C. 46
D. 45

Correct answer: D. 45
An antiderivative is F(x) = 2x^3 - 2x^2 + x + C. At x = 1 it gives 1 + C = 7, so C = 6. Substituting x = 3 gives 54 - 18 + 3 + 6 = 45.

8. What is the definite integral of 6x times (3x^2 + 2)^2 from x = 0 to x = 1?

A. 13
B. 125/3
C. 39
D. 117

Correct answer: C. 39
Use u = 3x^2 + 2, so du = 6x dx. The new bounds are 2 and 5. Evaluating the antiderivative u^3/3 at those bounds gives (125 - 8)/3 = 39.

9. A graph joins (0, 1), (1, 3), (2.5, 2), and (4, 5) in order using straight line segments. What is the exact integral of the graphed function from x = 0 to x = 4?

A. 11
B. 9.5
C. 10
D. 12

Correct answer: A. 11
Each trapezoid has area equal to its width times the mean of its two heights. The three areas are 2, 3.75, and 5.25. Their sum is 11.

10. What is the area between y = 2x and y = x^2 from x = 0 to x = 2?

A. 20/3 square units
B. 8/3 square units
C. 4 square units
D. 4/3 square units

Correct answer: D. 4/3 square units
The difference 2x - x^2 = x(2 - x) is nonnegative on [0, 2]. Its antiderivative is x^2 - x^3/3, giving 4 - 8/3 = 4/3.

11. Which is a general antiderivative of (x + 2)e^x, with C an arbitrary constant?

A. (x + 3)e^x + C
B. (x + 2)e^x + C
C. (x + 1)e^x + C
D. (x^2/2 + 2x)e^x + C

Correct answer: C. (x + 1)e^x + C
Using x + 2 as the factor to differentiate and e^x as the factor to integrate gives (x + 2)e^x - e^x + C, which simplifies to (x + 1)e^x + C.

12. What is the value of the improper integral of 6/x^4 from x = 2 to infinity?

A. 1/2
B. 1/4
C. -1/4
D. It diverges

Correct answer: B. 1/4
First integrate to a finite upper bound b. The antiderivative -2/x^3 gives 1/4 - 2/b^3. As b grows without bound, the second term tends to zero.

13. The average value of f(x) = a x^2 + 2 on [1, 3] is 9. What is the constant a?

A. 3/2
B. 7/13
C. 9/4
D. 21/13

Correct answer: D. 21/13
The integral of x^2 from 1 to 3 is 26/3. Dividing by the interval length 2 gives an average of 13/3. Solve 13a/3 + 2 = 9 to get a = 21/13.

14. For x from 0 to 4, a solid has square cross sections perpendicular to the x-axis. Each side length in units is 2 plus the square root of x. What is the volume?

A. 136/3 cubic units
B. 24 cubic units
C. 40/3 cubic units
D. 32 cubic units

Correct answer: A. 136/3 cubic units
The area of each square is (2 + sqrt(x))^2 = 4 + 4sqrt(x) + x. Integrating the terms from 0 to 4 gives 16 + 64/3 + 8 = 136/3 cubic units.

15. What is the exact integral of 3/((x + 1)(x + 4)) from x = 0 to x = 3? Here ln denotes the natural logarithm.

A. ln(7/16)
B. ln(4/7)
C. ln(16/7)
D. ln(7/4)

Correct answer: C. ln(16/7)
The integrand is 1/(x + 1) - 1/(x + 4). Its antiderivative on this interval is ln((x + 1)/(x + 4)). Subtract ln(1/4) from ln(4/7) to get ln(16/7).