Linear Programming Practice

Use this key to check answers and read the explanations.

1. A model uses P = 4x + 7y for a total score. The aim is to make P as large as possible. What is P = 4x + 7y called?

A. The objective function
B. A resource constraint
C. The feasible region
D. A corner point

Correct answer: A. The objective function
The objective function gives the score for each plan. The model seeks a feasible plan with the highest score.

2. A display holds x narrow items that take 2 cm each and y wide items that take 5 cm each. The items can use at most 35 cm. Which constraint models the combined width?

A. $2x + 5y \geq 35$
B. $5x + 2y \leq 35$
C. $2x + 5y \leq 35$
D. $x + y \leq 35$

Correct answer: C. $2x + 5y \leq 35$
The two item types use 2x and 5y centimeters. Their total width can be at most 35 cm.

3. A model accepts only nonnegative plans with $x + y \leq 7$ and $3x + 2y \leq 17$. Which plan (x, y) is feasible?

A. (5, 2)
B. (2, 4)
C. (0, 8)
D. (−1, 4)

Correct answer: B. (2, 4)
The plan (2, 4) gives 6 for the first sum and 14 for the second. The other plans respectively exceed the weighted limit, exceed the total limit, or use a negative value.

4. A feasible plan uses x = 2 and y = 5. What value does it give the objective P = 6x + 9y?

A. 63
B. 51
C. 48
D. 57

Correct answer: D. 57
Replace x with 2 and y with 5. The score is $6 \times 2 + 9 \times 5 = 57$.

5. A plan has limits $x + y \leq 9$ and $2x + 4y \leq 24$. At (2, 5), which limit is binding, meaning the plan reaches that limit exactly?

A. Only the limit of 9
B. Only the limit of 24
C. Both limits
D. Neither limit

Correct answer: B. Only the limit of 24
The first total is 7, leaving room under 9. The second total is 4 + 20 = 24, exactly at its limit.

6. A bounded feasible region has vertices (0, 0), (4, 0), (3, 2), and (0, 5). Which vertex maximizes $P = 5x + 4y$?

A. (3, 2)
B. (4, 0)
C. (0, 5)
D. (0, 0)

Correct answer: A. (3, 2)
The four corner scores are 0, 20, 23, and 20 in the order given. The highest occurs at (3, 2).

7. An outreach team can make at most 9 visits and use at most 25 staff-hours. Local visits x take 2 hours each and extended visits y take 3. Which plan uses both limits exactly?

A. (5, 5)
B. (7, 2)
C. (0, 8)
D. (2, 7)

Correct answer: D. (2, 7)
Set $x + y = 9$ and $2x + 3y = 25$. Subtracting twice the first equation from the second gives $y = 7$ and then $x = 2$.

8. An event needs at least 5 sessions, with at least 2 guided. A guided session costs 6 credits and an independent one costs 4. Which listed plan (guided, independent) meets the requirements at the lowest cost?

A. (3, 1)
B. (1, 4)
C. (2, 3)
D. (3, 2)

Correct answer: C. (2, 3)
The feasible choices are (2, 3) and (3, 2), which cost 24 and 26 credits. The other two choices miss a minimum requirement.

9. Both (2, 2) and (3, 1) are feasible. The objective is P = 8x + 5y. How does P change when a planner switches from (2, 2) to (3, 1)?

A. It increases by 3.
B. It decreases by 3.
C. It increases by 13.
D. It stays the same.

Correct answer: A. It increases by 3.
The switch adds one unit of x and removes one unit of y. The change in the objective is $8 - 5 = 3$.

10. A model has $x \geq 0$, $y \geq 0$, and $x + y \geq 4$. Its aim is to maximize $P = x + 2y$. Why does P have no finite maximum?

A. The largest feasible y is 4, and x can never change.
B. The constraints have no feasible plans.
C. Every feasible plan has a score of 4.
D. At x = 0, y can grow without limit and keep the plan feasible.

Correct answer: D. At x = 0, y can grow without limit and keep the plan feasible.
At x = 0, every $y \geq 4$ is feasible. Along that ray, $P = 2y$ grows without limit.

11. The feasible region is the polygon with corners (0, 0), (6, 0), (3, 3), and (0, 4), including its edges. Which set gives all the plans that maximize P = x + y?

A. The entire segment from (3, 3) to (0, 4)
B. Only the two points (6, 0) and (3, 3)
C. The entire segment from (6, 0) to (3, 3)
D. The entire feasible polygon

Correct answer: C. The entire segment from (6, 0) to (3, 3)
The highest corner value is 6 at (6, 0) and (3, 3). The edge between them has $x + y = 6$ everywhere. The other corners score below 6.

12. A media room has up to 10 hours and 35 credits. An hour of audio work uses 2 credits and earns 4 points. An hour of video work uses 5 credits and earns 7 points. With fractional hours allowed, what is the highest score?

A. 49 points
B. 55 points
C. 40 points
D. 70 points

Correct answer: B. 55 points
The two limits meet at 5 hours of each activity, for 55 points. Audio alone reaches 40 points, video alone reaches 49, and doing neither scores zero.

13. A fictional event uses x early and y late support hours, both nonnegative and allowed to be fractional. It must meet $x + 2y \geq 8$ and $3x + y \geq 9$. At $5 per early hour and $4 per late hour, which plan (x, y) minimizes cost?

A. (3, 2)
B. (8, 0)
C. (0, 9)
D. (2, 3)

Correct answer: D. (2, 3)
At each x, the least feasible y costs the least. The lower boundary has corners (8, 0), (2, 3), and (0, 9), costing $40, $22, and $36. Beyond (8, 0), more early hours only raise the cost.

14. A club chooses nonnegative whole numbers x and y with $3x + 2y \leq 14$ and $x + 2y \leq 8$. It wants to maximize $P = 7x + 6y$. Which plan (x, y) is best?

A. (4, 1)
B. (3, 3)
C. (3, 2)
D. (2, 3)

Correct answer: A. (4, 1)
For x = 0, 1, 2, 3, and 4, the largest feasible y is 4, 3, 3, 2, and 1, respectively. The resulting scores are 24, 25, 32, 33, and 34. Larger x is infeasible.

15. A display uses x and y hours in two modes and scores $P = 5x + 3y$. Hours are nonnegative and can be fractional. The limits are $x + y \leq 10$ and $3x + y \leq 18$. How much does the best score rise if 18 changes to 20?

A. 5 points
B. 4 points
C. 2 points
D. 10 points

Correct answer: C. 2 points
The old best corner is (4, 6), with 38 points. The new best corner is (5, 5), with 40 points. Both beat their respective axis corners, so the maximum rises by 2.