Polynomials Practice

Use this key to check answers and read the explanations.

1. What is the degree of $-6 + 4x^2 - x^5 + 3x$?
   A. 5
   B. 2
   C. 3
   D. 6
   Correct answer: A. 5
   The degree is the highest exponent with a nonzero coefficient. The term $-x^5$ supplies degree 5 even though it is written after other terms.

2. What is the coefficient of x^3 in $9x^5 - 2x^3 + x - 8$?
   A. -8
   B. 2
   C. -2
   D. 9
   Correct answer: C. -2
   The x^3 term is $-2x^3$, so the number multiplying x^3 is -2.

3. Which expression is a polynomial in x with real coefficients?
   A. $\sqrt{x} + 3x^2$
   B. $\sqrt{2} + 3x^2$
   C. $x^3 + 2/x$
   D. $x^2 + 3^x$
   Correct answer: B. $\sqrt{2} + 3x^2$
   A polynomial uses fixed nonnegative integer powers of x. The number $\sqrt{2}$ is a constant. In the other expressions, $\sqrt{x}$ uses a fractional power, $2/x$ uses a negative power, and 3^x puts x in the exponent.

4. If $P(x) = x^3 - 2x + 4$, what is $P(-2)$?
   A. 16
   B. -8
   C. 8
   D. 0
   Correct answer: D. 0
   Substituting -2 gives $(-2)^3 - 2(-2) + 4 = -8 + 4 + 4 = 0$.

5. What is $(4x^3 - 2x + 1) + (-x^3 + 5x - 6)$ after combining like terms?

A. $5x^3 + 3x - 5$
B. $3x^3 + 3x - 5$
C. $3x^3 + 7x + 7$
D. $3x^3 + 3x + 7$

Correct answer: B. $3x^3 + 3x - 5$
Combine the coefficients separately for x^3, x, and the constant. The sums are $4 - 1 = 3$, $-2 + 5 = 3$, and $1 - 6 = -5$.

6. What is $(3x^2 - x + 4) - (x^2 + 5x - 2)$ after combining like terms?

A. $2x^2 - 6x + 6$
B. $2x^2 + 4x + 2$
C. $2x^2 - 6x + 2$
D. $4x^2 - 6x + 6$

Correct answer: A. $2x^2 - 6x + 6$
Subtract every term in the second polynomial. This contributes $-x^2$, $-5x$, and $+2$, giving $2x^2 - 6x + 6$ after combining terms.

7. What is the expanded form of $-3x^2(2x^3 - x + 4)$?

A. $-6x^5 + 3x - 12$
B. $-6x^6 + 3x^3 - 12x^2$
C. $-6x^5 - 3x^3 - 12x^2$
D. $-6x^5 + 3x^3 - 12x^2$

Correct answer: D. $-6x^5 + 3x^3 - 12x^2$
Multiply $-3x^2$ by all three terms, adding exponents when multiplying powers of x. The middle term has a positive coefficient because the two minus signs cancel.

8. What is the expanded form of $(x^2 + 3)(2x - 5)$?

A. $2x^3 + 5x^2 + 6x - 15$
B. $2x^3 - 15$
C. $2x^3 - 5x^2 + 6x - 15$
D. $2x^3 - 5x^2 + 3x - 15$

Correct answer: C. $2x^3 - 5x^2 + 6x - 15$
The four products are x^2 times $2x$, x^2 times -5, 3 times $2x$, and 3 times -5. These give the four terms of the expanded expression.

9. For $x \neq 0$, what is $(8x^5 - 12x^3 + 4x^2)$ divided by $4x^2$?

A. $2x^3 - 3x + 1$
B. $2x^3 - 3x$
C. $2x^3 - 3x + 4$
D. $2x^7 - 3x^5 + x^4$

Correct answer: A. $2x^3 - 3x + 1$
Divide each coefficient by 4 and subtract 2 from each exponent. The last term gives $x^0 = 1$ because x is nonzero.

10. A rectangle is $(x + 6)$ meters by $(x + 2)$ meters. A rectangular hole measuring x meters by $(x - 2)$ meters is cut out. For $x > 2$, what polynomial gives the remaining area in square meters?

A. $8x + 12$
B. $6x + 12$
C. $2x^2 + 6x + 12$
D. $10x + 12$

Correct answer: D. $10x + 12$
The outer area is $x^2 + 8x + 12$ and the hole has area $x^2 - 2x$. Subtracting the hole cancels the x^2 terms and leaves $10x + 12$.

11. Let $P(x) = ax^3 + 4x^2 - 2$ and $Q(x) = 3x^3 - x^2 + 5x$. What value of a makes $P(x) + Q(x)$ have degree exactly 2?

A. 3
B. 0
C. -3
D. -4

Correct answer: C. -3
The x^3 coefficient in the sum is $a + 3$. Setting it to zero leaves $3x^2$ as the leading term because its coefficient is $4 - 1 = 3$.

12. Two polynomials each have degree 4. Their leading coefficients are 5 and -2. What are the degree and leading coefficient of their product?

A. Degree 8, leading coefficient 3
B. Degree 8, leading coefficient -10
C. Degree 16, leading coefficient -10
D. Degree 4, leading coefficient -10

Correct answer: B. Degree 8, leading coefficient -10
Multiplying the highest-degree terms gives $(5x^4)(-2x^4) = -10x^8$. Every other product has smaller degree, so this term determines both answers.

13. For $P(x) = -2x^6 + 30x^3 - 1$, what happens to $P(x)$ as x goes far to the left or far to the right?

A. It rises on the left and falls on the right.
B. It rises without bound on both ends.
C. It falls on the left and rises on the right.
D. It falls without bound on both ends.

Correct answer: D. It falls without bound on both ends.
The leading term $-2x^6$ dominates for large magnitudes of x. The sixth power is positive on either end, so its negative coefficient sends both ends downward.

14. What is the remainder when $2x^3 - x + 5$ is divided by $x + 2$?

A. -9
B. 19
C. -13
D. 5

Correct answer: A. -9
The remainder theorem evaluates the polynomial where the divisor equals zero. At $x = -2$, the value is $-16 + 2 + 5 = -9$.

15. A polynomial P has degree at most 3 and satisfies $P(-x) = P(x)$ for all real x. Also $P(0) = -1$ and $P(2) = 11$. Which formula fits all three conditions?

A. $2x^3 - x^2 - 1$
B. $x^3 + x^2 - 1$
C. $3x^2 - 1$
D. $3x^2 + 1$

Correct answer: C. $3x^2 - 1$
Replacing x with $-x$ changes the signs of the odd powers. Symmetry forces the x^3 and x coefficients to be zero. The constant is -1, and 4 times the x^2 coefficient minus 1 equals 11, so the coefficient is 3.