Vectors

Use this key to check answers and read the explanations.

1. Which description is a vector quantity?
   A. A displacement of 5 meters west
   B. A temperature of 20 degrees Celsius
   C. A mass of 5 kilograms
   D. An elapsed time of 5 seconds
   Correct answer: A. A displacement of 5 meters west
   Displacement has both a magnitude and a direction. The other measurements in the choices are scalars.

2. What movement is described by the component vector (3, −2) on a standard coordinate grid?
   A. Two right and three down
   B. Three left and two up
   C. Three right and two down
   D. Three right and two up
   Correct answer: C. Three right and two down
   The first component measures horizontal change and the second measures vertical change.

3. What is the magnitude of the vector (3, 4)?
   A. 7
   B. 5
   C. 12
   D. 25
   Correct answer: B. 5
   Its magnitude is the square root of $3^2 + 4^2$, or 5.

4. What is (2, 5) + (−5, 7)?
   A. (3, −12)
   B. (7, 2)
   C. (−7, 12)
   D. (−3, 12)
   Correct answer: D. (−3, 12)
   Add the horizontal components to obtain −3 and the vertical components to obtain 12.

5. Multiply the vector $(-2, 4)$ by the scalar 3. What is the result?

A. $(6, 12)$
B. $(-6, 12)$
C. $(-2, 12)$
D. $(1, 7)$

Correct answer: B. $(-6, 12)$
Multiply each of the two components by 3.

6. What is the displacement from $(-2, 3)$ to $(5, -1)$?

A. $(7, -4)$
B. $(3, 2)$
C. $(-7, 4)$
D. $(7, 2)$

Correct answer: A. $(7, -4)$
Subtract each starting coordinate from the corresponding ending coordinate. This gives $5 - (-2) = 7$ and $-1 - 3 = -4$.

7. A person walks 9 meters northeast and then 9 meters southwest along the same line. What is the resultant displacement?

A. 18 meters southwest
B. 18 meters northeast
C. 9 meters southwest
D. The zero vector

Correct answer: D. The zero vector
The two displacements have the same magnitude and opposite directions. Adding them gives zero displacement.

8. Which unit vector points in the same direction as $(-6, 8)$?

A. $(-3/4, 1)$
B. $(3/5, -4/5)$
C. $(-3/5, 4/5)$
D. $(-6/5, 8/5)$

Correct answer: C. $(-3/5, 4/5)$
The original magnitude is 10. Divide both components by 10 to obtain a vector of length one in the same direction.

9. What is the dot product of (4, −3) and (2, 5)?

A. −7
B. 23
C. 7
D. −23

Correct answer: A. −7
Multiply the corresponding components and add the products. The sum is 8 − 15 = −7.

10. A fictional object’s velocity is 7 meters per second east and 24 meters per second north. What is its speed?

A. 24 meters per second
B. 31 meters per second
C. 17 meters per second
D. 25 meters per second

Correct answer: D. 25 meters per second
The components are perpendicular. The speed is the square root of $7^2 + 24^2$, or 25 meters per second.

11. For what value of k are the vectors (6, 2) and (k, −9) perpendicular?

A. 9
B. −3
C. 3
D. 1/3

Correct answer: C. 3
The dot product must be zero. The equation 6k − 18 = 0 gives k = 3.

12. Which vector points in the same direction as (−2, 5)?

A. (4, −10)
B. (−4, 10)
C. (−4, 5)
D. (−10, 4)

Correct answer: B. (−4, 10)
A positive scalar multiple preserves direction. The vector (−4, 10) is twice the given vector.

13. Let u = (3, −4). Which vector v satisfies 2u + v = (11, −3)?

A. (5, −11)
B. (8, 1)
C. (17, −11)
D. (5, 5)

Correct answer: D. (5, 5)
Twice u is (6, −8). Subtract it from (11, −3) to get (5, 5).

14. An object moves 3 meters east. A constant force on it has components 12 newtons east and 5 newtons north. How much work does this force do over that displacement?

A. 36 joules
B. 39 joules
C. 51 joules
D. 15 joules

Correct answer: A. 36 joules
Work is the dot product of force (12, 5) and displacement (3, 0). The result is 12 × 3 + 5 × 0 = 36 joules.

15. Project u = (6, 2) onto v = (1, 1), then subtract that projection from u. What remains?

A. (−2, 2)
B. (4, 4)
C. (2, −2)
D. (6, 2)

Correct answer: C. (2, −2)
The projection coefficient is the dot product 8 divided by the squared length of v, which is 2. The projection is 4v = (4, 4). Subtraction leaves (2, −2).